

Évolution des microstructures et comportement élastoviscoplastique des matériaux

Approche phénoménologique par la théorie des champs de phase

Syllabus

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1 Diffusion et mécanique

1.1 Retour sur la thermodynamique de la diffusion

(de Groot and Mazur, 1984; Müller, 1985; Maugin, 1999) (Maugin, 2006) (Villani et al., 2014)

1.2 Couplage mécanique et diffusion

1.3 Une formulation du modèle de Cahn-Hilliard

Méthode des puissances virtuelles (Forest, 2018)

Les microforces et microcontraintes de Gurtin (Fried and Gurtin, 1993; Gurtin, 1996)

Lien avec la dérivation variationnelle (Cahn and Hilliard, 1958; Miehe, 2011)

1.4 Modèle de microdiffusion

(Forest, 2008) (Ubachs et al., 2004; Ubachs et al., 2005)

2 Changements de phase et mécanique

2.1 Une formulation du modèle de champs de phase

(Ammar et al., 2009a) (Gurtin, 1996)

2.2 Changements de phase et élasticité

microélasticité (Larché and Cahn, 1973; Larché and Cahn, 1978; Cahn and Larché, 1985; Fratzl et al., 1999)

calcul de l'effet de l'élasticité sur la concentration d'équilibre d'un alliage (Khachaturyan, 1983; Wang et al., 1993; Wang and Khachaturyan, 1995) (Dreyer and Müller, 2000; Dreyer and Müller, 2001)

2.3 Couplage à l'élastoviscoplasticité

(Gaubert et al., 2008; Ammar et al., 2009b; Gaubert et al., 2010; Finel et al., 2010; Cottura et al., 2016) (Ammar et al., 2011; Ammar et al., 2021) (Ubachs et al., 2006; Ubachs et al., 2007) (Borukhovich et al., 2014; Borukhovich et al., 2015)

2.4 Champs de phase et homogénéisation

Intervention de Sébastien Brisard (Steinbach and Apel, 2006; Steinbach, 2009; Steinbach and Shchyglo, 2011) (Ammar et al., 2009b; Rancourt et al., 2016)

(Mosler et al., 2014; Bartels and Mosler, 2017; Kiefer et al., 2017) excess stress (Durga et al., 2013; Durga et al., 2015; Chatterjee et al., 2022)

2.5 Applications récentes

applications récentes (Schmidt et al., 2021; Pinomaa et al., 2022; Lindroos et al., 2022)

2.6 Hérité de la déformation plastique et de l'écrouissage

(Ammar et al., 2014)

2.7 Plasticité à gradient et champs de phase

(Cottura et al., 2012) (Forest et al., 2011; Forest et al., 2014)

3 Un modèle micromorphe de l'endommagement et de la rupture

(Forest, 2009; Fandeur et al., 2022)

4 Milieu de Cosserat et champs de phase

Le modèle KWC de joint de grains (Kobayashi et al., 2000; Warren et al., 2003; Abrivard et al., 2012a; Abrivard et al., 2012b; Admal et al., 2019)

(Ask et al., 2018b; Ask et al., 2019; Ask et al., 2018a; Admal et al., 2018; Ask et al., 2020)

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FIGURE 1 – Bernard D. Coleman (1930-) à Paris (CMDS11, 2007), Morton E. Gurtin (1934-2022).



FIGURE 2 – Gordon school 1974 : John Werner Cahn (Cologne 1928-2016 Seattle, MIT-NIST), Francis C. Larché, André Pineau...



FIGURE 3 – Die 3 Wolfgangs : Wolfgang Müller, Wolfgang Muschik, Wolfgang Dreyer, 2003 (Institut für Mechanik, Institut für Theoretische Physik, Weierstraß-Institut, Berlin).



FIGURE 4 – La coopération avec Benoît Appolaire (ONERA puis Université de Lorraine) et Kais Ammar (Mines Paris).

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